Pricing Derivatives in Markets with Transaction Costs

Abstract

We model markets with proportional transaction costs using the methodology of indifference pricing. We use a general utility function to find the price of a European derivative to be given by a difference of solutions of two variational inequalities. We propose methods for solving those variational inequalities.

Markets with transaction costs

Virtually every market has a different price for the sell and the purchase of an asset. This is commonly referred as a bid-ask spread. Some reasons for the existence of spreads are trading costs or the illiquidity of the asset. One approach to explain the spread purchase of an asset. This is commonly referred as a bid-ask spread. Transaction Costs

We model markets with proportional transaction costs using the methodology of indifference pricing. We use a general utility function to find the price of a European derivative to be given by $\Delta = (x + y - z) - (x + y - z)$.

Pricing via utility maximisation

We assume that the agent can choose between two different portfolios, where both are comprised by stocks and money but one of them has a short position on a financial derivative portfolio. We also consider that the investor would find the trading strategies $\{L, M\}$ which yield him an optimal terminal utility. Therefore the present value of both portfolios is given by,

$$V_f(x, y, s, t) = \sup_{L,M} \mathbb{E}_{x,y,s}[U(X_T + Y_T S_T)]$$

$$v(x, y, s, t) = \sup_{L,M} \mathbb{E}_{x,y,s}[U(X_T + Y_T S_T - C_T)]$$

where $X_T$ denotes the amount of money in risk-free assets, $Y_T$ de number of shares, $L_T$ the cumulative number of shares bought and $M_T$ the cumulative number of shares sold. Introducing the risk-premiums $z$ and $z'$ given by

$$U(x + y - z') = v_f(x, y, s, t)$$

$$U(x + y - z) = v(x, y, s, t)$$

one finds that the price of the financial derivative is given by $p_c = z - z'$. This approach is referred in the literature as utility indifference pricing [2].

From the dynamic programming principle [3] one finds that the risk premiums $z$ and $z'$ are solutions of the following variational inequality

$$\min\left\{z_t + \frac{\sigma^2 s^2}{2} \left(R(x + y s - z)(y - z_s)^2 + z_{ss}\right) - (y - z_s)\alpha s
\right.$$ \nonumber

$$, -(s - z_s) - s(1 + \mu)z_x
\right.$$ \nonumber

$$, (s - z_s) + s(1 - \mu)z_x \right\} = 0$$

where the terminal conditions are, respectively,

$$z(T, x, y, s) = c_T \quad z'(T, x, y, s) = 0.$$ 

We note that in equation (3) we consider a general risk-aversion $R(t) := -\frac{U'(t)}{U''(t)}$, which is an enhancement to existing models [1].

Outlook

The main challenges with this problem lie in solving the variational inequality (3). Next developments will be solving the problem numerically using methods such as the penalty method and front-fixing method.

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References


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