Fitted Operator Methods and Special Meshes in Computational Finance

Abstract

We consider option pricing in a market which experiences instances of liquidity and illiquidity. The market takes the form of a regime-switching continuous two state Markov process. We study the investor’s problem of maximising both terminal wealth and option payoff whose solution can be found as a solution of a semilinear coupled Hamilton-Jacobi-Bellman equation.

Models

In most modern option pricing, models are characterised by discontinuous payoff functions and volatility of underlying may be very low. Risk preferences in optimal investment models may be expressed by variables which vanish in the limit. Against this background, numeric solutions are sought that are adapted to resolve the resulting degenerate equations.

We consider is a system of ODEs which arises in option pricing for markets that switch between liquid (state 0) and illiquid (state 1):

\[
\begin{align*}
\gamma R_0^i - \frac{1}{2} \sigma^2 S^2 R_{SS}^i &= -\nu_0 e^{-\gamma R_0^i} e^{R_0^i} + d_0 + \nu_{01}, \\
\gamma R_1^i &= -\nu_1 e^{-\gamma R_1^i} e^{R_1^i} + d_1,
\end{align*}
\]

with the terminal conditions \( R_i(T, S) = h(S), \ i = 0, 1 \). The aim is to develop positive preserving, stable numerical schemes. We use an implicit approximation for the two equations and linearise the exponential term using Taylor expansion.

Another way of looking at this system is recasting it as a Partial Integro-Differential Equation (PIDE):

\[
\psi_t - \frac{1}{2} \sigma^2 S^2 \psi_{SS} = -ae^{\psi(S,t)} \left( c \int_0^t e^{-\psi(S,s)} ds + e^{-\psi(S,t)} \right) + b - c, \\
\psi(S,0) = \gamma h(S).
\]

The PIDE can be approximated using implicit in time together with a quadrature method for the integral term. A difficulty of this PIDE is that the upper limit of the integral is nonconstant. We use a trapezoidal rule for our quadrature rule together with a Taylor expansion linearisation of the nonlinear term.

Results

Fig. 1 shows the approximate solution to \( R^0 \) which are the dynamics of the price when the market is liquid. As is expected, the surface takes the form of the Black-Scholes surface where the market is assumed to be always liquid.

In Fig. 2 we compare the value of the option in the liquid state at maturity and issue of the option. As expected, the option value increases with time to maturity.

Dissemination

The outcomes of the project so far include presentations and papers that have been submitted for conference proceedings at:

- 40th International Conference Applications of Mathematics in Engineering and Economics (AMEE14), Sozopol, Bulgaria, June, 2014.

Figure 1: Solution surface of \( R^0 \).

Figure 2: Comparing European option values at issue and maturity in the liquid state.