Option Pricing in Exponential Lévy Models with Transaction Costs

Abstract

In this work we want to extend the approach of Davis, Panas and Zariphopoulou for the option pricing problem in presence of proportional transaction costs, considering that the market prices follow an exponential Lévy process.

Introduction

In a market with transaction costs, there is no portfolio that replicates an European call option. Moreover, the super-replication in the presence of transaction costs turns out to be too costly. In the paper [1], the authors use a different definition for the price of the option based on the utility function approach of Hodges and Neuberger [2]. With this method, the problem of finding the price of the option, turns out to be a pair of stochastic optimal control problems with different terminal conditions. In this model, the dynamics of the underlying asset follows a geometric Brownian motion. It is well known that this is an unrealistic representation of the real market dynamics. We replace this process with a more general exponential Lévy model. Using the stochastic control theory we obtain the Hamilton-Jacobi-Bellman (HJB) equation which is a partial integro-differential equation (PIDE) written as quasi-variational inequality.

Controlled portfolio dynamics

Suppose an investor has a portfolio with two kind of securities:

- Risk free assets: B (amount of money in bank account)
- Risk assets : Y (number of shares)

The portfolio dynamics is:

\[
\begin{align*}
    dB(t) &= rB(t)dt - (1 + \lambda)S(t)dL(t) + (1 - \mu)S(t)dM(t) \\
    dY(t) &= dL(t) - dM(t) \\
    S(t) &= S(0)e^X_t
\end{align*}
\]

The two processes \(L(t)\) and \(M(t)\) are the cumulative number of shares bought or sold in \([0,T]\). The constants \(\lambda\) and \(\mu\) are the proportional transaction costs. So the bank account \(B(t)\) and number of shares \(Y(t)\), change according to the two strategies \(L(t)\) and \(M(t)\).

The price follows a geometric Lévy process, where \(X_t\) is a Lévy process with characteristic triplet \((\alpha, \sigma^2, \nu)\).

Option pricing

Define the cash value:

\[
c(y, s) = \begin{cases} 
    (1 + \lambda)sy, & \text{if } y < 0 \\
    (1 - \mu)sy, & \text{if } y > 0 
\end{cases}
\]

These are the residual cash value when long position are sold and short position are closed.

We consider two utility maximisation problems: maximising the wealth of a portfolio with just cash and shares, and another portfolio with an additional option \(p\). The value functions are:

\[
\begin{align*}
    V_1(t, b, y, s) &= \sup_{L, M} \mathbb{E}^{b, y, s}[U(B(T) + c(Y(T), S(T)))] \\
    V_2(t, b + p, y, s) &= \sup_{L, M} \mathbb{E}^{b + p, y, s}[U(B(T) + p(T) + I_{S \leq K})c(Y(T), S(T)) \\
    &+ I_{S > K}(K + c(Y(T) - 1, S(T)))]
\end{align*}
\]

Where \(U : \mathbb{R} \rightarrow \mathbb{R}\) is a concave increasing utility function. We will use the exponential utility: \(U(x) = 1 - e^{-\gamma x}\)

Sample path of controlled process

The controls force the process to stay inside the no transaction region (red lines). In this case the free boundaries are straight lines, because the utility function is: \(U(x) = \frac{x^2}{\gamma}\)

Solution of the problem

The price of the option can be found implicitly by \(V_1(t, b, y, s) = V_2(t, b + p, y, s)\). We need to solve numerically the HJB equation:

\[
\begin{align*}
    \max \{ & V_t + rbV_b + \alpha sV_s + \frac{1}{2} \sigma^2 s^2 V_{ss} + \\
    & \int_{\mathbb{R}} [V(t, b, y, s + z) - V(t, b, y, s)] \frac{\partial V}{\partial s} \nu(dz) , \\
    & V_b -(1 + \lambda)sV_b, -V_y -(1 - \mu)sV_b \} = 0
\end{align*}
\]

Future developments:

- Solve the problem using Markov chain approximation as described in [3].
- Studying the theory of viscosity solution for PIDEs.
- Compare the numerical results with other methods: penalty method and splitting method.
References


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