EXAMPLE OF DYNAMIC PROGRAMMING ALGORITHM FOR THE TSP

Distance matrix:

$$C = \begin{pmatrix} 0 & 2 & 9 & 10 \\ 1 & 0 & 6 & 4 \\ 15 & 7 & 0 & 8 \\ 6 & 3 & 12 & 0 \end{pmatrix}$$

$g(2, \emptyset) = c_{21} = 1$
$g(3, \emptyset) = c_{31} = 15$
$g(4, \emptyset) = c_{41} = 6$

k = 1, consider sets of 1 element:

Set {2}: $g(3,\{2\}) = c_{32} + g(2, \emptyset) = c_{32} + c_{21} = 7 + 1 = 8$ $p(3,\{2\}) = 2$
$g(4,\{2\}) = c_{42} + g(2, \emptyset) = c_{42} + c_{21} = 3 + 1 = 4$ $p(4,\{2\}) = 2$

Set {3}: $g(2,\{3\}) = c_{23} + g(3, \emptyset) = c_{23} + c_{31} = 6 + 15 = 21$ $p(2,\{3\}) = 3$
$g(4,\{3\}) = c_{43} + g(3, \emptyset) = c_{43} + c_{31} = 12 + 15 = 27$ $p(4,\{3\}) = 3$

Set {4}: $g(2,\{4\}) = c_{24} + g(4, \emptyset) = c_{24} + c_{41} = 4 + 6 = 10$ $p(2,\{4\}) = 4$
$g(3,\{4\}) = c_{34} + g(4, \emptyset) = c_{34} + c_{41} = 8 + 6 = 14$ $p(3,\{4\}) = 4$

k = 2, consider sets of 2 elements:

Set {2,3}: $g(4,\{2,3\}) = \min \{c_{42} + g(2,\{3\}), c_{43} + g(3,\{2\})\} = \min \{3+21, 12+8\} = \min \{24, 20\} = 20$
$p(4,\{2,3\}) = 3$

Set {2,4}: $g(3,\{2,4\}) = \min \{c_{32} + g(2,\{4\}), c_{34} + g(4,\{2\})\} = \min \{7+10, 8+4\} = \min \{17, 12\} = 12$
$p(3,\{2,4\}) = 4$

Set {3,4}: $g(2,\{3,4\}) = \min \{c_{23} + g(3,\{4\}), c_{24} + g(4,\{3\})\} = \min \{6+14, 4+27\} = \min \{20, 31\} = 20$
$p(2,\{3,4\}) = 3$

Length of an optimal tour:

$f = g(1,\{2,3,4\}) = \min \{ c_{12} + g(2,\{3,4\}), c_{13} + g(3,\{2,4\}), c_{14} + g(4,\{2,3\}) \}$
$= \min \{2 + 20, 9 + 12, 10 + 20\} = \min \{22, 21, 30\} = 21$

Successor of node 1: $p(1,\{2,3,4\}) = 3$
Successor of node 3: $p(3, \{2,4\}) = 4$
Successor of node 4: $p(4, \{2\}) = 2$

Optimal TSP tour: $1 \rightarrow 3 \rightarrow 4 \rightarrow 2 \rightarrow 1$